Team Control Number

13014

Problem Chosen

A

2022

HiMCM/MidMCM

Summary Sheet

Honeybees are crucial to the survival of humanity. These insects fertilize the crops that feed billions of people worldwide. Given the significance of these insects' role and the recent decline of honeybee populations, it is critical to understand trends in bee population over time, determine which factors have the most significant impact on a honeybee colony's population, and model the number of hives needed to pollinate a given field.

To this end, we developed a bee population model that predicts the population of worker and drone bees in a colony at a given time. We constructed differential equations to model the change in worker and drone bee populations. To create these differential equations, we considered the following factors: the initial population of bees, ratio of total bees to drone bees, length of the active honeybee season, egg laying rate, average lifespan of bees, and gestational period of bees. Next, we utilized Euler's method to predict these trends over ten years. We determined that the population of a honeybee colony ranges from approximately 19,046 bees at the end of the inactive season to approximately 60,759 bees at the end of the active season. The worker bee population ranges from 19,046 to 60,053 bees, and the drone bee population ranges from 0 to 706 bees. These results are validated by online sources about typical bee populations ("Honey Bee Colony," 2021). We observed that the cycle of bee population repeats sinusoidally on a yearly period, so we also performed a sinusoidal regression, allowing farmers to estimate bee populations in the future.

When performing sensitivity analysis on the population model, we modified each factor considered by up to 40% in either direction. This analysis revealed that the egg-laying rate, worker bee lifespan during the active/inactive season, and length of the active season had the greatest impact on the population of bees over ten years. In contrast, modifying the initial population, the ratio of total bees to drone bees, drone lifespan, or any gestational period had little to no effect on the bee population.

We also created a model that utilizes the bee population model to predict the number of hives needed to pollinate a 20-acre field. In the pollination model, we consider the field area, the area required by each flower, the number of flowers pollinated by each worker bee, and the maximum distance a bee travels from its hive (called "bee range"). By calculating the total number of flowers on the field and the number of flowers worker bees pollinate daily, we discovered that it would be best to use 22 hives to pollinate a 20-acre field. Since we discovered the placements of hives did not matter, these 22 hives are assumed to all be placed at the center of the 20-acre field. This result was also confirmed by the University of Georgia, which recommends using approximately one beehive for every acre that needs pollination ("Managing bees for pollination," n.d.). However, using 21 or 20 can also have similar effects, and using one or two fewer hives results in less money spent on buying hives and maintenance for a minimal gain in pollination.

To assess the robustness of our model, we performed a sensitivity analysis on all four factors considered. By modifying each factor up to 40% in either direction, we determined that changing the honeybee range had the most significant effect on the number of hives needed. A greater bee range resulted in more hives needed. Decreasing the flowers visited per day per bee and decreasing the area needed per flower also increased the number of hives required. In contrast, modifying the field size by up to 40% had no impact on the number of hives needed to pollinate the field. Since bees typically stay within 6,000 m of their hive, the 20-acre field is relatively tiny compared to the circle with a radius of 6,000 m pollinated by the hive. Thus, whenever the side length of the field is as small as 285 m, changing the field's size by a relatively small amount has no impact on the number of hives needed to pollinate the field.

Lastly, we created an infographic that outlines all of our findings and relevant information for the general public. We hope this will be a valuable resource for the general public and policymakers to better understand the populations of honeybee colonies and their ability to fertilize crops within a given area.

Table of Contents

Modeling Honeybee Population

Problem Restatement

In the first problem, we are tasked with developing a model that accurately predicts the population of a honeybee colony over time. We were provided with basic information about a honeybee's habits:

1. Honeybees can travel up to 20 km but typically stay within 6 km of their hive.
2. A typical honeybee hive contains between 20,000 and 80,000 honeybees.
3. A single honeybee can visit approximately 2,000 flowers or more in a single day.
4. Because of the high workload during summertime, most honeybees work themselves to death, resulting in a shorter lifespan.
5. During autumn and wintertime, honeybees may live a bit longer (four to six months).
6. A honeybee's level of activity, pollen consumption, and protein abundance impact its lifespan.

Assumptions

Assumption 1.1: *The ratio of total bees to drone bees stays relatively constant throughout the active season (April 1st → September 15th).*

- Justification: Without direct observation of bee populations in a hive, it is virtually impossible to identify the exact ratio of total bees to drone bees. However, many sources online (Grad, 2010) cite a consistent ratio between drone bees and total bees.

Assumption 1.2: *The length of the honeybee's active season stays relatively constant every year.*

- Justification: While the actual honeybee season depends on the temperature and seasonal patterns of flowers, it is difficult to account for these inconsistencies ("When are bees most active?," 2022). Additionally, over long periods of time, the length of the honeybee's active season will likely stay relatively constant.

Assumption 1.3: *A queen honeybee produces eggs at a relatively constant rate per day*

- Justification: It is impossible to account for the specific factors that influence the number of eggs laid by the queen bee per day. However, over long periods of time, the number of eggs laid by the queen bee per day is likely to stay relatively constant.

Assumption 1.4: *The average lifespan of bees (both drone and worker) stays relatively constant during the active season and is at a lower but still constant level during the inactive season.*

- Justification: It is virtually impossible to identify the exact lifespan of a bee, as the conditions in which honeybees die, day by day, are unpredictable. Thus, we assume that honeybees die at a constant rate per day. Additionally, honeybees are less active during autumn and wintertime, as stated in the problem, so we assume that the honeybees die at a lower constant rate per day during these seasons.

Assumption 1.5: *The average gestational periods of both drone and worker bees stay relatively constant throughout the year.*

- Justification: Identifying the exact times when certain bees undergo gestation is challenging. However, most drone bees have similar gestational periods, and most worker bees have similar gestational periods.

Assumption 1.6: *The measurements are approximately normally distributed.*
- Justification: We assume the given measurements are average values. By the Central Limit Theorem, a series of averages follow a normal distribution.

Variables

Variable	**Definition**	**Default Value**	**StdDev**	**Units**
$P_{initial}$	(Factor) Initial population of bees in the honeybee colony	40,000 Bees; Average population of honeybee hive (given from problem)	N/A	Bees
N	(Factor) Number of total bees for 1 drone bee to appear	101 bees; 100 workers for 1 drone bee, so 101 total bees for 1 drone bee (Grad, 2010)	5 Bees	Bees
λ	Fertilization rate of eggs, or the chance of an egg developing into a worker bee. Calculated as: $\lambda = 1 - \frac{1}{N}$	$\lambda = 1 - \frac{1}{101} = 99\%$	N/A	N/A
L_{season}	(Factor) Length of the active honeybee season	167 Days; Days from April 1 → September 15; length of honeybee active season ("When does Honey Bee Season Start?," 2018), ("The Beekeepers Year," n.d.)	10 Days	Days
E_{new}	(Factor) Number of eggs laid per day by the queen honeybee	1,500 Eggs ("The colony and its organization," n.d.)	300 Eggs	Eggs
L_{worker}	(Factor) Average lifespan of a worker bee during the active honeybee season (Summer)	42 Days ("Honey Bee Life Span," 2021)	3 Days	Days
L_{drone}	(Factor) Average lifespan of drone bees during the active honeybee season	60 Days (Rueppell et al., 2005)	3 Days	Days

I_{worker}	(Factor) Average lifespan of a worker bee during the inactive honeybee season	175 Days (Remolina & Hughes, 2008)	5 Days	Days
I_{drone}	(Factor) Average lifespan of a drone bee during the inactive honeybee season	10 Days; Drone bees are "kicked out" of the hive when winter starts, so they die quickly (Linnell & Marski, 2020)	1 Day	Days
G_{worker}	(Factor) Average gestational period of a worker bee	21 Days; ("Information sheet 3," n.d.)	3 Days	Days
G_{drone}	(Factor) Average gestational period of a drone bee	24 Days; ("Honey Bee Life Cycle," 2021)	3 Days	Days
t	Current Day	Varies during simulation	N/A	Days
$P(t)$	Population of the beehive at time t.	Varies during simulation	N/A	Bees
$W(t)$	Population of worker bees at any given time (t)	Varies during simulation	N/A	Bees
$D(t)$	Population of drone bees at any given time (t)	Varies during simulation	N/A	Bees

Table 1: Variables along with their respective definitions and units

Model Development

In modeling the population of a honeybee hive, we consider a honeybee colony located in a field with typical amounts of flowers, pollen, nectar, predators, and other factors. The population of a honeybee colony is divided into three types of bees: (1) queen bees, whose role is to lay eggs; (2) worker bees (majority of bees), whose role is to maintain the colony's welfare; (3) drone bees, whose role is to fertilize the virgin queen ("The colony and its organization," n.d.). Since there is only one queen bee in a honeybee colony, we only monitor the population of drone bees and worker bees in a honeybee colony over time.

When developing our model, we consider the factors marked as "(Factor)" in Table 1. Each factor has a default value and its standard deviation. With each day that passes, we account for variations in the value of each factor by randomly choosing a value from a normal distribution that is based on the mean and standard deviation of the factor. Note that a different value for initial population is not modified, as that is only used once at the beginning of the simulation. The default value acts as the mean, while the value from the "StdDev" column is the value chosen that day. This is done to account for day-to-day errors in the default value.

Simulating the Active Honeybee Season

The simulation begins at the start of the active honeybee season, typically April 1st, which we denote as $t = 0$. Since fertilized eggs will turn into worker bees and unfertilized eggs will turn into drone bees, we can assume that the probability of any honeybee being a worker bee will be λ and the probability of any honeybee being a drone bee will be $1 - \lambda$. Thus, initially, there are $P_{initial}$ number of bees, with a $W = P_{initial} \cdot \lambda$ number of worker bees (W) and $D = P_{initial} \cdot (1 - \lambda)$ number of drone bees (D). With each day that passes, we account for changes caused by five factors: the queen bee laying new eggs, worker bees dying, drone bees dying, worker bees gestating, and drone bees gestating. We constructed three differential equations to model these changes in eggs, development, and deaths during the active honeybee season.

Suppose the number of eggs at time t, where t is a day during the active season, is modeled by the function $E(t)$. Using Euler's method with a step size of $\Delta t = 1$, $E(t + 1) = E(t) + \frac{dE}{dt}$, where $E(0) = 0$. The differential equation $\frac{dE}{dt}$, which represents the change in eggs per day (the step size), is modeled by: $\frac{dE}{dt} = E_{new} - ((E\frac{1}{G_{worker}} \cdot \lambda) + (E\frac{1}{G_{drone}} \cdot (1 - \lambda)))$. Essentially, the change in eggs, $\frac{dE}{dt}$, is the change in the number of eggs laid by the queen (which become honeybees, represented by E_{new}) minus the number of eggs that finish gestation. The number of days it takes for worker and drone bees to gestate is represented by G_{worker} and G_{drone}, respectively. Approximating over a long period of time, we expect that each day, $\frac{1}{G_{worker}}$ of the fertilized eggs (which become worker bees, represented by $E \cdot \lambda$) and $\frac{1}{G_{drone}}$ of the unfertilized eggs (represented by $E \cdot (1 - \lambda)$) finish gestation. Therefore, the change in eggs, $\frac{dE}{dt}$, is E_{new} minus $E\frac{1}{G_{worker}} \cdot \lambda + E\frac{1}{G_{drone}} \cdot (1 - \lambda)$.

Next, suppose the population of a type of bee (b, either a worker or drone bee) is given be $B(t)$. Using Euler's method, Using Euler's method with a step size of $\Delta t = 1$, $B(t + 1) = B(t) + \frac{dB}{dt}$, where $B(0) = P_{initial} \cdot C$, where C is the probability that any given bee is type b. The differential equation $\frac{dB}{dt}$, which represents the change in the number of bees per day, is modeled by: $\frac{dB}{dt} = \frac{1}{G} \cdot C \cdot E - \frac{1}{L_{worker}} \cdot B$, where G is the gestation period for that type of bee in days. Essentially, the change in the number of bees, $\frac{dB}{dt}$, is the number of eggs that finish gestation minus the number of bees that die that day. Approximating over a long period of time, we expect that each day, $\frac{1}{G}$ of the eggs of type b (represented by $C \cdot E$) will gestate. Thus, on any given day, $\frac{1}{G} \cdot C \cdot E$ eggs of type b finish gestation. As we approximate over the long-term, $\frac{1}{L}$ of type b bees can be expected die on any given day, where L is the lifespan of the bee in days. Since there are B type b bees, $\frac{1}{L}B$ of them die on any given day.

Therefore, the change in worker bees, which have a gestation period of G_{worker}, lifespan of L_{worker}, and probability of gestating from an egg of λ, is: $\frac{dW}{dt} = \frac{1}{G_{worker}} \cdot \lambda \cdot E - \frac{1}{L_{worker}} \cdot W$, as W is the total population of worker bees at a given time. By the same logic, the change in drone bees, which have a gestation period of G_{drone}, lifespan of L_{drone}, and probability that any bee is a drone bee of $(1 - \lambda)$, is: $\frac{dD}{dt} = \frac{1}{G_{drone}} \cdot (1 - \lambda) \cdot E - \frac{1}{L_{drone}} \cdot D$, as D is the total population of drone bees. These differential equations only apply for the length of the active season (167 days).

Simulating the Inactive Honeybee Season

During the inactive season, from September 16 to March 31, the queen lays no new eggs (Kiley, 2017), and drone bees are kicked out of the hive and left to die (Linnell & Marski, 2020). Therefore, during the inactive season, the change in eggs is $\frac{dE}{dt} = 0$. The change in worker bees is $\frac{dW}{dt} =- \frac{1}{I_{worker}} \cdot W$. Essentially, the number of worker bees that die per day is represented by the reciprocal of a worker bee's winter lifespan, multiplied by the approximate number of total worker bees. The same logic is applied to the change in drone bees, $\frac{dD}{dt}$, to obtain $\frac{dD}{dt} =- \frac{1}{I_{drone}} \cdot D$. These differential equations only apply for the non-active season, parts of the year other than the active season.

These results can be summarized in Table 2.

Equations	**Use When**
$\frac{dE}{dt} = E_{new} - ((E\frac{1}{G_{worker}} \cdot \lambda) + (E\frac{1}{G_{drone}} \cdot (1 - \lambda)))$ $\frac{dW}{dt} = \frac{1}{G_{worker}} \cdot \lambda \cdot E - \frac{1}{L_{worker}} \cdot W$ $\frac{dD}{dt} = \frac{1}{G_{drone}} \cdot (1 - \lambda) \cdot E - \frac{1}{L_{drone}} \cdot D$	$0 \leq t \leq 167$, where t = mod(t) 365; while $t > 365$
$\frac{dE}{dt} = 0$ $\frac{dW}{dt} =- \frac{1}{I_{worker}} \cdot W$ $\frac{dD}{dt} =- \frac{1}{I_{drone}} \cdot D$	All other times

Table 2: Piecewise differential equations for our model. Note that $E(t)$, $W(t)$, and $D(t)$ remain the same year-round.

Results & Analysis

Utilizing the default values from Table 1, and the methods described above, we modeled the population of a honeybee colony over a 10-year period, as shown in Figure 1.

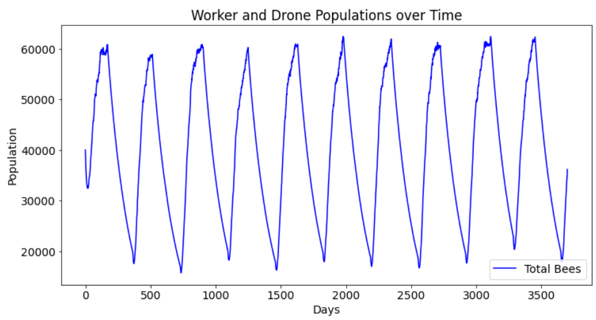


Figure 1: Population of honeybee hive over a 10-year period.

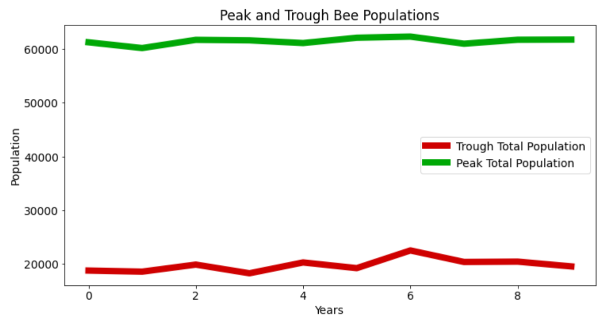


Figure 2: Peak and trough of honeybee population over a 10-year period.

Figure 1 shows a honeybee colony's population over a ten-year period. Additionally, Figure 1 appears to be a periodic function, which intuitively makes sense, as the lifespan of bees and birth rate of eggs all vary on a yearly period. In order to quantify the bee population over this 10 year period, we extracted the peak—which occurs at the end of the active season—and trough—which occurs at the end of the inactive season—populations over the ten year period. The peak and troughs over 10 years can be seen in Figure 2.

Since the peak and trough lines are relatively constant, we used the average of the peaks and troughs over the 10 year period for each year as a measure of bee population. The average trough population was 19,046 bees, and the average peak population was 60,759 bees. The problem statement, which states that the typical honeybee hive contains between 20,000 to 80,000 honeybees, validates our result. The percent difference between the average trough bee count and 20,000 is 4.9%, which may have been due to minor differences in our default values in Table 1 and the real world.

Additionally, we collected data on the populations of worker and drone bees individually over the 10 year period. The populations of workers and drones over 10 years can be seen in Figure 3. As expected, the population of worker bees is drastically higher than drones, because there is usually 1 drone bee for 100 worker bees ("The colony and its organization," n.d.). Thus, worker bees are the major determining factor in the population of a honeybee colony. We also track the populations of workers and drones in their peaks and troughs, as shown in Figure 4. The average peak of workers was 60,053 bees, and the average peak of drones was 706 bees. The average trough of workers was 19,046 bees, and the average trough of drones was 0 bees (occurs during winter).

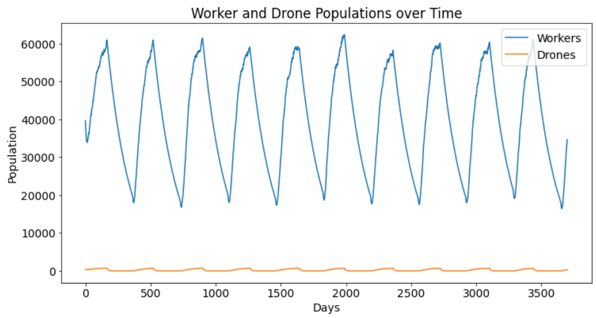


Figure 3: Population of workers and drones over a 10 year period.

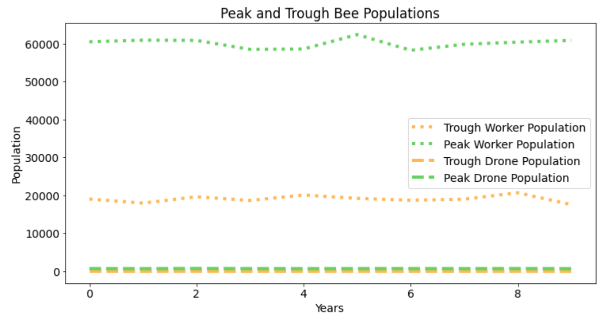


Figure 4: Peak and trough of workers and drones over a 10 year period.

Sinusoidal Regression

Due to the periodic repeating nature of the population in Figure 1, we also performed a sinusoidal regression. The best-fit parameters were a midpoint of 40,022, an amplitude of 18,648, period of 366, and a displacement of 57 in the positive direction.

Thus, the best fit sine curve is: $P(t) = 18648 \cdot sin(\frac{2\pi}{366} \cdot (t - 57)) + 40022$, where $P(t)$ is the beehive's population at time t (measured in days). This equation produced an R^2 value of 0.951, which essentially means that the sinusoidal equation models the population of bees over time very well (Frost, 2022). The original data and the sinusoidal regression is shown in Figure 5.

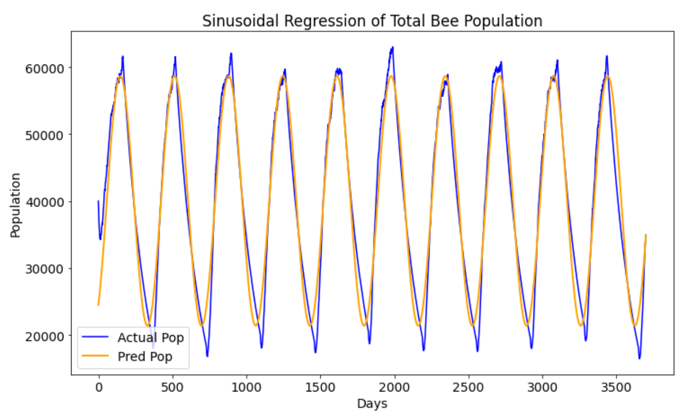


Figure 5: Sinusoidal regression using scikit-learn on the total population data.

Sensitivity Analysis

In order to test the robustness of our model, we performed sensitivity analysis by varying all factors in Table 1 (initial bee population, ratio of total bees to drone bees, length of active season, number of eggs laid per day, lifespan of bees during active/inactive seasons, and gestational period of bees). Each factor in Table 1 was both increased and decreased by 10%, 20%, 30%, and 40% for a total of 8 changes to each factor total. For each factor changed, all other factors stayed constant. To determine the effect of changing these factors, we measured the average peak and trough of drone and worker bee populations over 10 years.

Most Significant Effect

Changing the egg-laying rate (E_{new}) had the biggest effect on the bee population. The effect of changing E_{new} is shown in Figure 6. Since the average population varies slightly between runs, due to accounting for error with standard deviation, we ran each modification 10 times and averaged the values for each metric across the ten runs. For example, we increased the egg laying rate by 30%, and ran it 10 times with this modification, and averaged the metrics across each run for TroughTotal (average total bee population at troughs), TroughWorker (average worker bee population at troughs), TroughDrone (average drone bee population at troughs), PeakTotal (average total bee population at peaks), PeakWorker (average worker bee population at peaks), and PeakDrone (average drone bee population at peaks) to produce one data point for 30% increase in E_{new}.

In Figure 6, each data point's color represents the amount by which the factor was changed by, with the green dot representing the original value. Each metric is on the x-axis, and bee-population is on the y-axis, so the greater the range of points on the y-axis for each metric, the greater the impact by the factor.

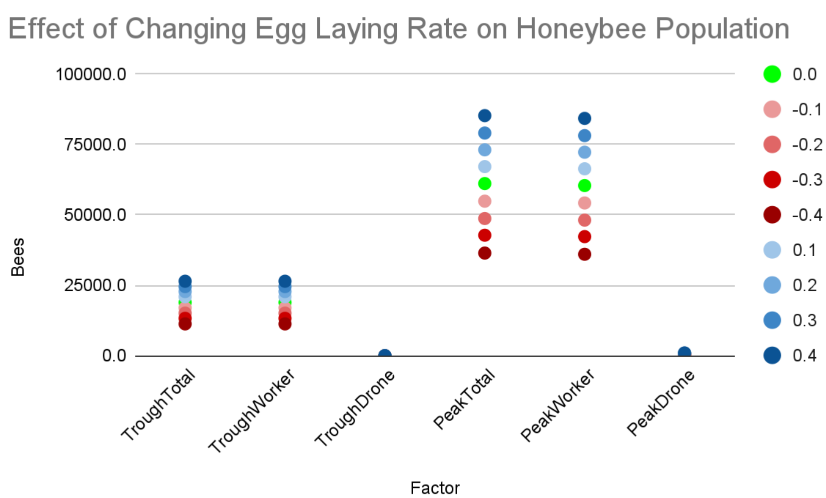


Figure 6: Changing the queen's egg laying rate (E_{new}) had the largest impact on bee population. The different colored dots represent how much egg laying rate was changed by and their respective values for each metric.

The factor that had the largest effect on the populations was the egg laying rate. This makes sense as the egg laying rate is the main driver of population growth during the active season. As expected, the higher the egg laying rate, the higher the honeybee population.

The factors with the second and third largest effects on bee populations were a worker bee's average lifespan during the active season and a worker bee's average lifespan during the inactive season, respectively. This also makes sense because the average lifespan of worker bees is the only factor that decreases population size. The effects of modifying these factors can be seen in Figure 7 and Figure 8.

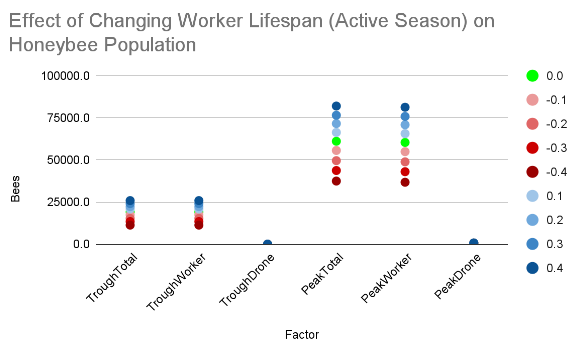


Figure 7: Changing worker lifespan during the active season (L_{worker}) had the 2nd largest impact on bee population.

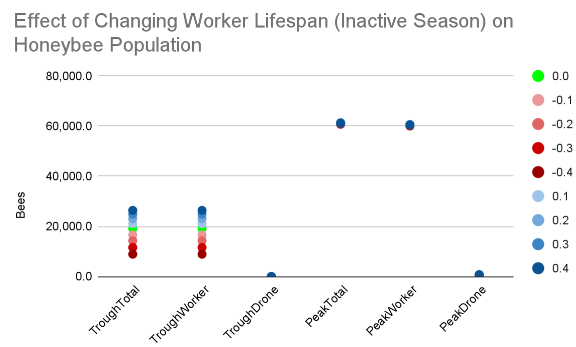


Figure 8: Changing worker lifespan during the inactive season (I_{worker}) had the 2nd largest impact on bee population.

Additionally, changing the length of the active season (L_{season}) also had a large impact on the honeybee colony's population. Increasing the length of the active season affects the worker population more than the drone population, as the drone population is relatively much smaller than the worker bee population. As such, the number of eggs produced affects the total number of worker bees more. Increasing the length of the active season results in an increase in the bee population, because the queen bee has more time to lay eggs before needing to stop for the winter. Effects of changing the length of the active season can be seen in Figure 9.

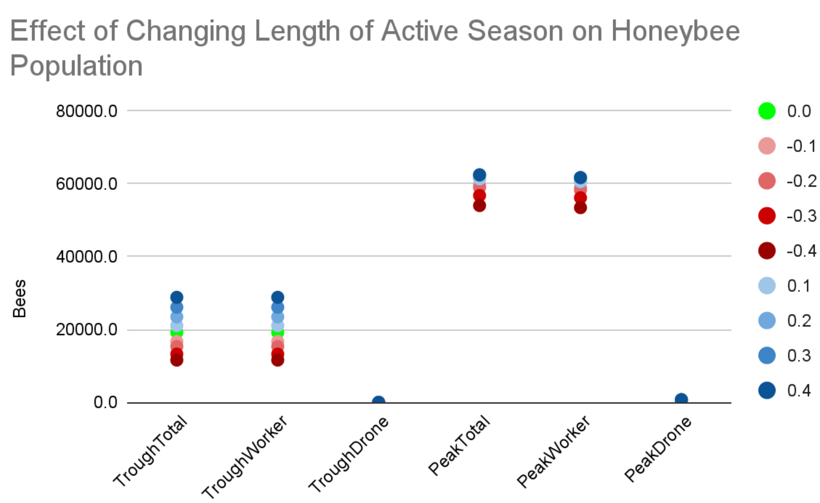


Figure 9: Changing the length of the active season has a larger impact on bee population than changing the length of the bee's lifespan during the inactive season.

Least Significant Effect

While the egg laying rate, worker bee lifespan (active and inactive seasons), and length of the active season had the greatest impacts on a bee's population, 6 factors had minimal or no effect on bee population. Among these are initial population ($P_{initial}$), the ratio of total bees to drone bees (N), drone lifespan in the inactive season (I_{drone}), drone lifespan in the active season (L_{drone}), gestational period of worker bees (G_{worker}), and gestational period of drone bees (G_{drone}).

For these factors, their impact was so minimal that it's almost impossible to discern each data point within a metric, showing a smaller range of the points along the y-axis for each metric. An example of the initial bee population's modifications can be seen in Figure 10. Modifying initial population had little effect, which makes sense, because the bee's population already periodically alternates between peak and troughs.

When modifying a drone's lifespan in both the inactive and active seasons, there was almost no effect on the bee population. This is because in comparison to the total bee population, the number of drones is so small that a quicker death has relatively no impact on the overall population. Modifying a drone's lifespan during the active season in shown in Figure 11.

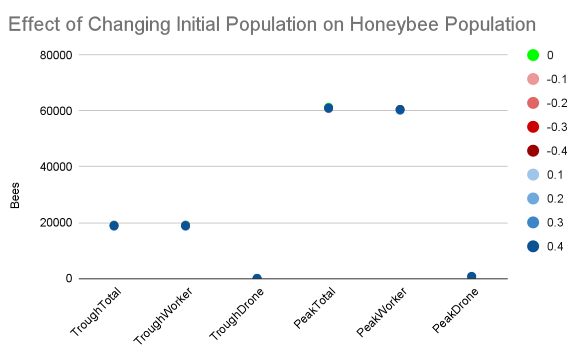


Figure 10: Changing the initial population had minimal impact on the final bee population.

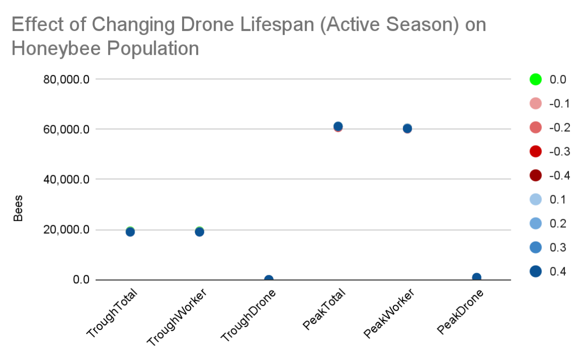


Figure 11: Changing the drone lifespan (active season) has almost no impact on honeybee population.

Additionally, the gestational period of both worker and drone bees had no impact, resulting in graphs similar to Figures 10 and 11. This is likely because since the egg laying rate stays the same, regardless of the gestational period, almost the same number of bees will eventually be produced.

Strengths and Weaknesses

Strengths
- The model can easily be adjusted to simulate longer or shorter amounts of time.
- The model accounts for errors in the

Weaknesses
- Our model assumes that many factors remain relatively constant throughout the active and inactive season, which may not be the case.

default values found from online using random values chosen from a normal distribution.
- Our model's results match up with the information given by the problem on how a honeybee hive typically contains between 20,000 to 80,000 bees.
- There were no reference values for each factor's standard deviation, so they were produced using inferences.

Pollination of a 20-Acre Field

Problem Restatement

In this problem, we are tasked with determining the number of honeybee hives needed to support pollination of a 20-acre parcel of land containing crops that benefit from pollination. We are provided with the same basic information about bees as in the "Modeling Honeybee Population" section.

Assumptions

Assumption 2.1: *Crops are placed with even spacing in the horizontal and vertical directions on the 20-acre field.*

- Justification: Each crop requires a certain amount of land to grow. In order to maximize the number of crops on a 20-acre field, we space the crops with the same minimal vertical and horizontal spacing needed for each crop. Thus, any other field layout with more spacing could still use the same number of hives predicted by our model.

Assumption 2.2: *A given honeybee will not travel outside a 6-kilometer radius from the hive.*

- Justification: The problem states that bees typically stay within 6 km of their hive. Since it is impossible to know how many bees travel farther than the 6 km, we assume that all bees stay within the 6 km.

Assumption 2.3: *The bees are evenly distributed throughout the 6 km circle around the hive, meaning that they evenly pollinate the crops within the 6 km radius.*

- Justification: Since we know that the bees will typically stay within 6 km of the hive, it is reasonable to assume that they are evenly distributed within this space. We have no data about the distribution of the bees within the possible 6 km they can travel from the hive.

Assumption 2.4: *Each honeybee hive in the field will have the same population at a given time.*

- Justification: Since all of these honeybee hives contain the same species of honeybee and operate under similar environmental conditions, we will assume that all of the hives have the same population at a given time, as it is impossible to know each hive's individual population.

Variables

Variable	Definition	Default Value	Units
A_{field}	(Factor) Field Area	81,000 m^2 (given from problem)	Square Meters (m^2)
A_{flower}	(Factor) Flower Area	0.023 m^2 (sunflowers are spaced 6 inches apart, so each flower takes up 36 in^2 of space). 36 in^2 = 0.023 m^2 ("The Ultimate Guide to Growing Sunflowers," 2020)	Square Meters (m^2)
D	(Factor) Daily flowers pollinated by bee	5,000 flowers ("How many flowers can a bee pollinate?," n.d.)	Flowers
R	(Factor) Bee range. Max distance a bee travels from its hive.	6,000 m; given by problem	Meters (m)
$P_{worker}(t)$	Function of worker bee population of one hive over time. Calculated by multiplying the total population by the fertilization rate. $P_{worker}(t) = P(t) \cdot \lambda$.	Calculated from previous model	Worker bees
$P_{total}(t)$	Function of total number of worker bees at a given time. Calculated by multiplying the population of workers per hive by the number of hives. $P_{total}(t) = P_{worker}(t) \cdot H$	Varies based on time	Worker Bees
L_{season}	Length of the active honeybee season	167 Days; Days from April 1 → September 15; length of honeybee active season ("When does Honey Bee Season Start?," 2018), ("The Beekeepers Year," n.d.)	Days
H	Number of hives	N/A	Hives
S	Scoring Metric	N/A	NA

$F_{density}$	Number of flowers pollinated in one square meter of the field or hive	N/A	Flowers / square meter
F_{total}	Number of flowers pollinated per day by the hives.	N/A	Flowers
$F_{pollinated}$	Number of flowers that are pollinated in the field	N/A	Flowers
γ	Percent of flowers pollinated every day	N/A	Percent

Table 3: Variables, definitions, their units, and default values used. Note that variables from Table 1 wil be used in this problem as well.

Model Development

When modeling the number of hives needed to pollinate a 20-acre field, we consider a square field with an area of 20-acres, or 81,000 m^2. Since the field is a square, the side length of this field is $\sqrt{81000\ m^2} = 284.6\ m$. Additionally, from the problem description, we are given that honeybees typically stay within 6,000 m of their hive. For simplicity, we assume that the bees are evenly pollinating the flowers within this 6,000 m. Figure 12 shows the area of one beehive compared to size of the field.

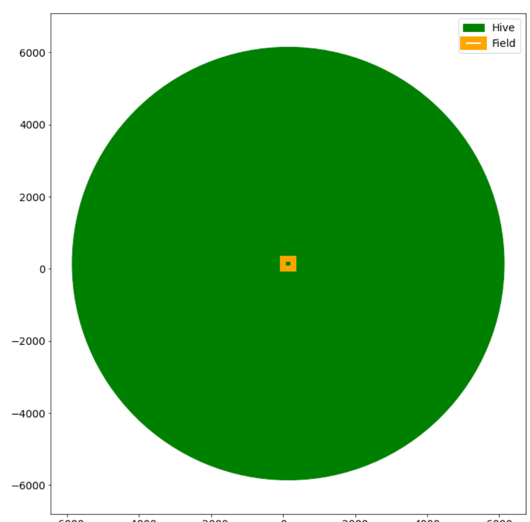


Figure 12: Size of the field compared to the coverage of a beehive when bees typically stay within 6,000 m of the hive. All axis labels are in meters.

From Figure 12, it is clear that the coverage of the beehive is far greater than the size of the field. However, it is important to note that this does not mean only one hive is enough to pollinate the entire field. Since bees will spread out evenly across the 6 km radius around the hive, many plants pollinated will be outside of the 20-acre field that needs to be pollinated. Thus, only a small fraction of the crops pollinated will be in the 20-acre field. Since the coverage of the hive is far greater than the size of the field, the placement of the hive within the field does not matter, as the hive will cover the entire field.

When developing our model to determine the number of hives needed to pollinate a 20-acre field, we considered four main factors: field area (A_{field}), the area needed by each flower (A_{flower}), the number of flowers pollinated per bee each day (D), and the typical max distance a bee travels from its hive (R). These variables and their default values are shown in Table 3. Additionally, our model is built off of the population model from the previous problem, so we also utilize the factors in Table 1.

Our model builds off the bee population model and predicts the percentage of plants on the 20-acre field that are pollinated on any given day. The number of hives that produces a result where the average number of plants pollinated per day is 100% is called the best model. Similar to the bee population model, the simulation begins at the start of the honeybee active season (April 1st), which we call $t = 0$. Recall from the bee population model that $P(t)$ represents the population of a honeybee colony at time t, where t is the number of days since the beginning of the active honeybee season.

Since we are assuming that all of the hives in the 20-acre field have the same population at a given point in time (t), to get the total number of worker bees that pollinate the field ($P_{total}(t)$), we multiply the number of worker bees per hive with the number of hives:
$P_{total}(t) = P_{worker}(t) \cdot H$.

Additionally, we know that every bee visits approximately D flowers every day, so the total number of flowers pollinated in a day by all worker bees (F_{total}) is $F_{total} = P_{total}(t) \cdot D$.

Moreover, to determine the number of flowers pollinated in each square meter of the hive coverage ($F_{density}$), we divide the total number of flowers pollinated by the bees by the total coverage of a hive: $F_{density} = \frac{F_{total}}{\pi R^2}$. Since the hive coverage encapsulates the entire field, the density of flowers on the field is also $F_{density}$.

Since we know the density of flowers in the 20-acre field, we can calculate the total number of flowers pollinated within the field per day ($F_{pollinated}$) by multiplying $F_{density}$ by the area of the field. Thus, we have: $F_{pollinated} = F_{density} \cdot A_{field}$.

To find the percentage of flowers pollinated by the bees in the entire field, we also need to calculate the total number of flowers in the field. We assume all crops in the field are sunflowers, which take up the same amount of space each. Therefore, to calculate the total number of flowers in the field (F_{field}), we divide the area of the 20-acre field (A_{field}) by the area used by each flower (A_{flower}): $F_{field} = \frac{A_{field}}{A_{flower}}$.

Thus, to calculate the percentage of flowers pollinated for a given day ($P_{pollinated}$), we divide the number of flowers pollinated by bees ($F_{pollinated}$) by the total number of flowers in the field (F_{field}). Thus, $\gamma = \frac{F_{pollinated}}{F_{field}} = \frac{P_{total}(t) \cdot H \cdot D \cdot A_{field}}{\pi R^2} \cdot \frac{A_{flower}}{A_{field}} = \frac{P_{total}(t) \cdot H \cdot D \cdot A_{flower}}{\pi R^2}$. An important note is that the area of the field is canceled out and has no effect on the percent of the field pollinated by the hives. This makes sense because regardless of the field size, the amount of flowers the bees will pollinate per square meter in the field will remain the same as long as the field size is smaller than the maximum range of the bees. We realize that this assumption causes our model to break when the field size is larger than the maximum range.

We utilize this model and iterate through the days up until 10 years, and record the percent of flowers pollinated per day (γ), as long as t is during the active season. This is because worker bees do not pollinate flowers during the winter ("Do bees still make honey during the winter?," 2022). We calculate the average of the percents in γ to determine the average percentage of flowers pollinated per day ($\overline{\gamma}$). Since sunflowers must attract a pollinator in less than 2 days, we want the average percentage of flowers pollinated per day to be as close to 100% (represented by "1") as possible ("Sunflowers and Bees," 2018). Thus, we score the hive configurations using the following metric: $Score = |\overline{\gamma} - 1|$. Choosing the number of hives that produces the lowest $Score$ is the most optimal number of hives.

Results & Analysis

By utilizing the methods in the above section, we determined that 22 bee hives performed the best for pollinating a 20-acre field. The spacing of these 22 hives can be seen in Figure SPACING. However, using 21 and 20 hives was also very effective at pollinating the field. These results are consistent with the University of Georgia's recommendation for one beehive per acre for pollination ("Managing bees for pollination," n.d.).

Utilizing the scoring method outlined above, the values in Table 4 were produced, with lower scores representing a model that's better at pollinating the 20-acre field.

Number of Hives	**Score**
22	0.021
21	0.026
20	0.072
19	0.118

18	0.165

Table 4: The most optimal number of hives to pollinate a 20-acre field.

However, it is important to note that since each honeybee hive costs around \$120-\$200 (Harris, 2019), it may be a better choice to proceed with using 21 or 20 hives, as the minimal improvement with score costs up to \$400 more.

In fact, when graphing the score achieved by the number of hives used, the graph in Figure 13 is produced.

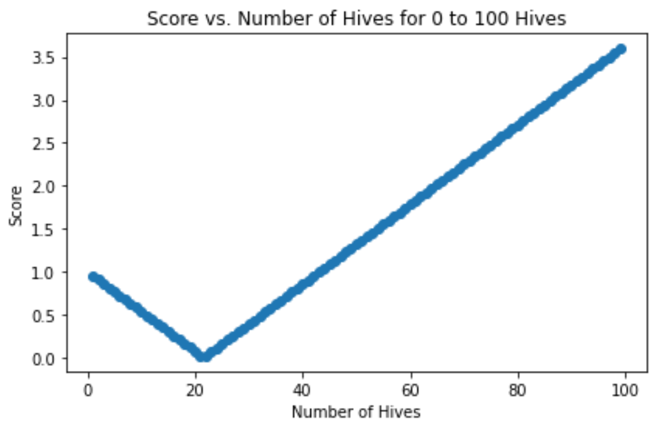


Figure 13: Score vs. Number of Hives when hives range from 0 to 100.

The graph in Figure 13 resembles an absolute value function. When fitting an equation to this function, we see that $Score = |0.0464(x - 22) + 0.021|$. Thus, with every hive that's added, the score increases by 0.0464.

Additionally, we measured the percentage of flowers pollinated on the field every day over a 10 year period, as shown in Figure 14.

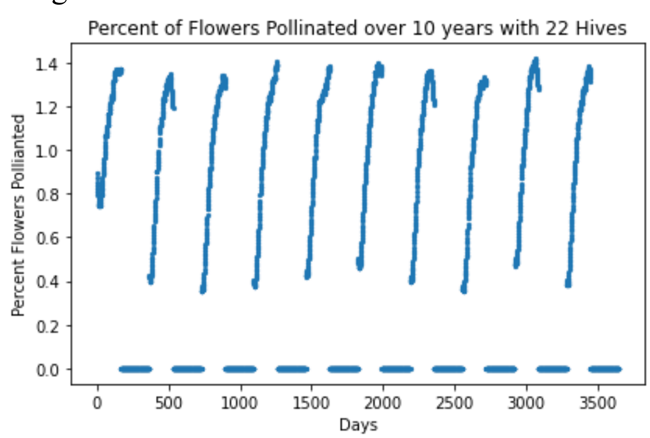


Figure 14: Percent of flowers on the field pollinated over a 10-day period. Another way to think about this graph is the average number of bees that pollinate a single flower in a given day

From Figure 14, we see that the percentage of flowers pollinated increases during a honeybee's active season (because the population of workers are increasing), as expected, but drops to zero during the inactive season, which is when worker bees huddle around the queen to keep warm, instead of pollinating. We also see that at the end of a honeybee's active season, the percentage of flowers on the field that's pollinated reaches a peak of around 1.4%. This means that some crops are pollinated twice. Note that this is not extreme enough to cause over-pollination ("Flowers damaged by too many bee visits," 2016).

Sensitivity Analysis

Additionally, we perform sensitivity analysis to determine the factors that have the greatest impact on the hives needed to pollinate 20 acres of a field filled with crops. We increase and decrease each factor considered by 10%, 20%, 30%, and 40%. We determined that the factors that had the largest impact on the number of hives needed to pollinate a 20-acre field were the distance a bee can travel from the hive, the number of flowers visited per day by a bee, and the area needed by each sunflower.

Most Significant Effect

When changing the range a honeybee typically stays in, more honeybee hives are needed, as the same number of bees are pollinating a larger area (which falls outside of the 20-acre field). This means fewer bees will be pollinating flowers in the 20-acre field, resulting in more hives needed to pollinate flowers within the field. Figure 15 shows the effect of changing the bee range on the number of hives needed to pollinate a 20-acre field. Note that the best, the 2nd best, and 3rd best number of hives to use are shown.

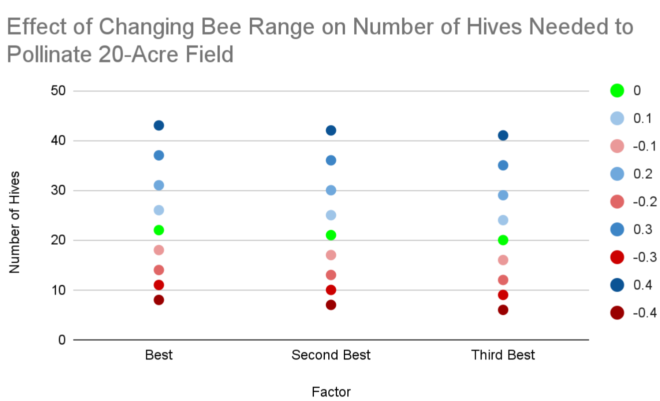


Figure 15: Changing the distance a bee flies from the hive has the greatest impact on the optimal number of hives.

Figure 15 clearly shows a smaller range of values for both the best, second best, and third best number of hives to use than when changing the typical distance a bee flies from the hive.

Additionally, changing the number of flowers visited per bee each day and changing the area needed by each flower are shown in Figures 16 and 17.

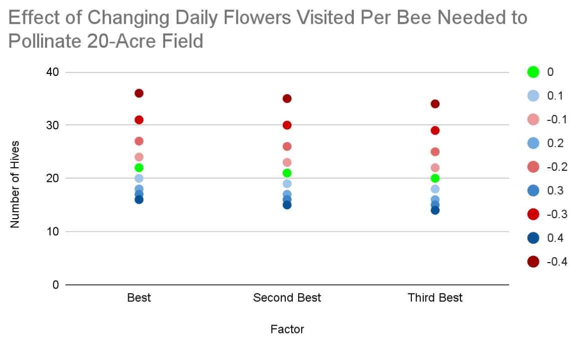


Figure 16: Changing the daily flowers visited per bee drastically changed the number of hives needed.

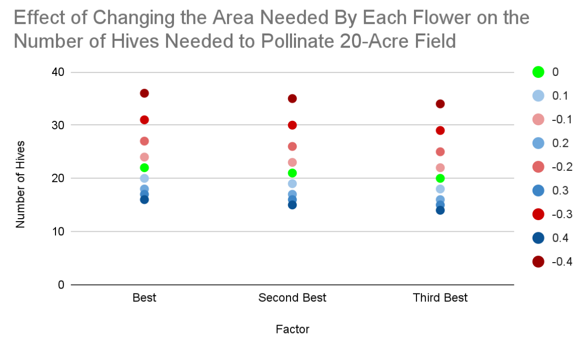


Figure 17: Changing the area needed by each flower drastically changed the number of hives needed.

From Figure 16, we see that increasing the number of flowers visited by each bee daily decreases the number of beehives needed. Intuitively, this makes sense, as each bee visiting more flowers means more flowers can be pollinated with the same number of bees.

From Figure 17, we see that increasing the area needed by each flower decreases the number of beehives needed. Intuitively, this also makes sense because there are fewer flowers to pollinate in the 20-acre field.

Least Significant Effect

In contrast, changing the field area seemed to have a minimal impact on the number of hives needed. Changing the field area and its impact on the number of hives needed to pollinate the field is shown in Figure 18.

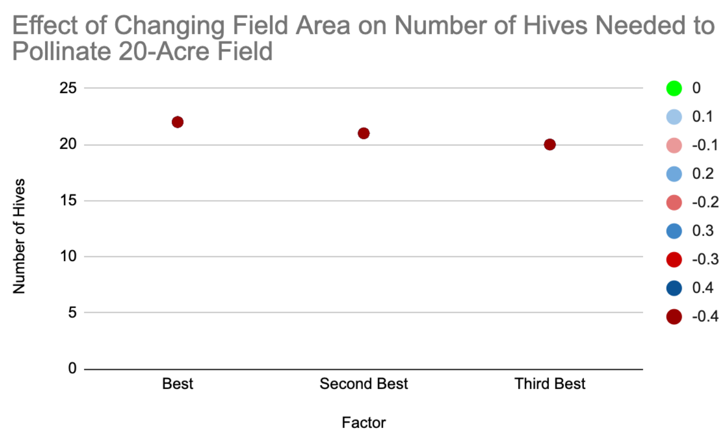


Figure 18: Effect of changing the field area on the number of hives needed to pollinate that field.

This is a surprising result, as one would expect that increasing the field area results in more hives needed to pollinate the entire field. However, recall that our model does not depend on the area of the field (when the field is relatively much smaller than the hive's coverage area), since the coverage area of each hive is so much larger than the 20-acre field's area.

Strengths and Weaknesses

Strengths

- Our model's result confirms online validation that approximately one honeybee hive is needed per acre of the field for pollination ("Managing bees for pollination," n.d.).
- It is easy to use our model to visualize all the different possibilities of hives and their scores, no matter the range of hive numbers to test.
- Our model only utilizes the worker honeybee population when determining how many flowers get pollinated. This is accurate to the real world, where worker bees pollinate flowers.

Weaknesses

- Our model assumes that bees are evenly distributed across the 6 km radius coverage of a hive. However, this may not be true, because there is a 20-acre field filled with flowers near the hive, so bees will tend to concentrate in the field more.
- Using our model, the entire worker bee population is outside of the hive pollinating. However, this may not be the case in the real world, as a certain number of worker bees likely stay in the hive and carry out hive responsibilities without going out for pollination.

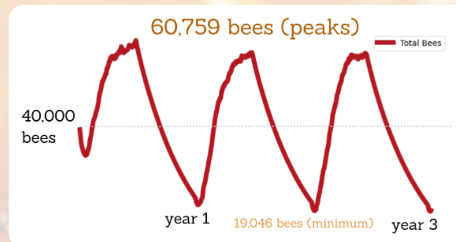
Non-Technical Infographic

Bee Smarter

------------- A guide to flowering goodness -------------

60,759
average hive peak population (from April → September)

19,046
average hive minimum population (from September → March)



bee population varies from ~19,000 → ~60,000 in yearly cycles

At peak populations:

- 60,053 worker bees
- 706 drone bees
- *99% of bees are workers*

At minimum populations:

- 19,046 worker bees
- 0 drone bees
- *Drone bees all die in the winter*

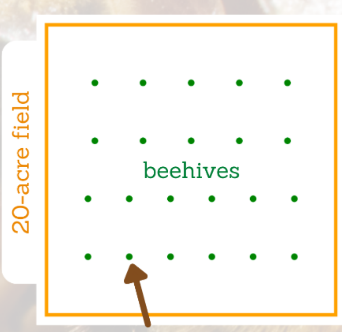
Top factors that affect bee population

Egg laying rate (1,500 eggs/day)
Higher egg laying rate = higher population

Worker bee lifespan (42 days, active season)
Longer active season lifespan = higher population

Length of active season (167 days)
Longer active honeybee season = higher population

22
beehives needed to pollinate a 20-acre field with 3,487,500 crops



Placement of hives does not matter!

Top factors that affect number of beehives needed to pollinate a field

Bee range (6 km)
Longer range = more hives needed (less flowers pollinated in field)

 # Flowers visited per day (5,000 flowers)
More flowers visited per bee / day = less hives needed

Area taken up per flower (0.023 m^2)
More space taken up per flower = less hives needed

Conclusion

To answer the given problem, we developed two models: one for simulating a honeybee colony's population over time and another to determine how many honeybee hives are needed to pollinate a 20-acre field.

In our honeybee population model, we created three differential equations to individually model important components contributing to changes in the bee population: number of eggs, worker bees, and drone bees. By considering the factors of the initial population, ratio of total bees to drone bees, length of the active honeybee season, number of eggs laid by the queen per day, average lifespan of bees, and average gestational period, we modeled the population of a honeybee colony over a 10-year period. To enhance the accuracy of our model, we analyzed the populations of worker and drone bees separately. The population of a honeybee colony seems to follow a sinusoidal pattern, with the population increasing through the active season and decreasing through the inactive season. The average peak population over 10 years was 60,759 bees, while the average minimum population was 19,046 bees, which occurs during the winter. The strength of this population model lies in its flexibility in simulating various lengths of time and its ability to account for errors in the default values found through online research. Furthermore, the results from the model match up with the given population range in the problem. However, our model does have a few limitations as it does assume that several factors—such as the ratio of total bees to drone bees, gestational period, death rates, and egg-laying rates—remain constant throughout the active and inactive seasons. In addition, we had to infer the standard deviation for each factor due to a lack of data on the topic.

Furthermore, we developed another model to determine the optimal number of honeybee hives needed to pollinate a 20-acre field. We consider the field area, the area needed by each flower, the number of flowers pollinated by each worker bee, and the max distance a bee travels from its hive. We determined that using 22 hives was the most optimal in pollinating the 20-acre field, but using 21 and 20 hives also provided acceptable results with potentially lower upkeep costs.

We also conducted sensitivity analysis on both models. The egg-laying rate and worker bee lifespan significantly impacted the honeybee population model. The bee range, the number of flowers each bee visited per day, and the area occupied by each flower greatly impacted the number of hives needed to pollinate the 20-acre field. Interestingly, modifying the field size with relatively small changes did not affect the number of hives needed to pollinate the field.

In the future, we would like to incorporate gradual changes in the average lifespan of bees to more realistically represent the changing nature of bee population across seasons than the sudden changes between active & inactive seasons in our current model. Moreover, we would also like to incorporate more factors into our model—such as adding nectar collection and reserves into our honeybee population model. Additionally, we would like to build onto the pollination model to allow inputs of a field's length, width, bee density, and flower density to output the number of hives needed to pollinate that field. By deploying such a model as a website using HTML, CSS, and JavaScript, farmers could easily utilize our model in the real world.

By pollinating the crops that feed the 7 billion people around the world, bees are an essential part of nature. With our model, farmers can estimate the population of a honeybee colony over time and determine the most optimal number of hives to pollinate a 20-acre field.

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